

Multi-objective optimization of Stirling engine using non-ideal adiabatic method



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ABSTRACT

In the recent years, remarkable attention is drawn to Stirling engine due to noticeable advantages, for instance a lot of resources such as biomass, fossil fuels and solar energy can be applied as heat source. Great numbers of studies are conducted on Stirling engines and non-ideal adiabatic method is one of them. In the present study, the efficiency and the power loss due to pressure drop into the heat exchangers are optimized for a Stirling system using non-ideal adiabatic analysis and the second-version Non-dominated Sorting Genetic Algorithm. The optimized answers are chosen from the results using three decision-making methods. The applied methods were compared at last and the best results were obtained for the technique for order preference by similarity to ideal solution decision making method.

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1. Introduction

Increasing the price of fossil fuel, environmental pollution, and noise pollution due to the engines using this fuel, has caused additional motivation for research on other types of power generation. High thermal efficiency, minimal pollution, reliability, maximum utilization and clean combustion are the major features that we expect from the new engines. In the recent years, the Stirling engines have attracted a lot of attentions due to high capability [1,2]. A Stirling engine presents a reasonable theoretical efficiency which can be closer to the Carnot efficiency, comparing with other reciprocating thermal engines [3]. According to the Stirling engine's ability to achieve the highest possible efficiency, this engine always is noticed for the researchers. With this regard, the first classical analysis of Stirling engine was done by Schmidt [4]. Schmidt assumed that the gas is isothermal in the expansion and the compression spaces. This assumption reduces the complexity of the compression and the expansion spaces with considering constant temperatures in the spaces.

In the real engines, during the process, the working spaces tend to the adiabatic mode mostly. Based on this fact, Finkelstein carried out the first analysis of non-constant temperature of the Stirling engine in 1960. In the analysis presented by him, each part of the engine (cooler, heater, regenerator, expansion and compression spaces) was considered as a control volume and the conservation laws of mass and energy were analyzed using the equation of state [5]. Martini provided computer simulation of Stirling engine with

considering five working spaces for it [6]. In this model, pressure drop and heat loss were intended. Also, Urieli and Berchowitz used a computer code for solving differential equations using Runge–Kutta fourth-order method. They presented an adiabatic model for non-ideal mode in order to improve the numerical predictions. In this method, the concept of the non-ideal regenerator was realized [7].

Abbas et al. developed a non-ideal adiabatic method. In their work, the regenerator space was divided into two parts and this model included losses such as the shuttle loss and regenerator loss [8]. Strauss and Dobson developed other model using the quasi-adiabatic analysis in which the losses of the regenerator were considered [9]. Granados et al. developed the quasi-steady flow model to calculate the pressure drop through the heat exchanger channels. In their model, the engine was divided into 19 parts which 10 control volumes were dedicated to the regenerator [10]. Tlili et al. developed the second adiabatic model based on the quasi-steady flow. In their modeling approach, they considered the shuttle losses, the internal and external conduction losses in the regenerator, and the loss of energy that lead to pressure drop in the heat exchangers [11]. Parlak et al. carried out the thermal analysis on the Stirling engine with the Gamma structure. The quasi-steady flow analysis was performed to achieve more precise results. The thermal efficiency reached to 25% using nitrogen as working fluid under pressure of 6.5 bar and the heat source temperature of 873 K [12]. Considering the different methods of optimization in the energy issues, which have been carried out recently, it is worth to do multi-objective optimization of which, some are explained as followings.

Different engineering problems such as skyline computation and vehicle routing issues have applied multi-objective

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Nomenclature

Q_h	heat transfer to heater	M	total mass of the working fluid
Q_k	heat transfer to cooler	m	mass of the working fluid in the different parts of the engine
Q_r	heat transfer to regenerator	Re	Reynolds number
p	mean effective pressure	Pr	Prandtl number
S	stroke	St	Stanton number
f_r	engine's frequency	$f(x)$	objectives function
T_h	temperature of the working fluid of hot side	X	vector of decision variables
T_H	temperature of the heat source		
T_c	temperature of the heat sink		
W_c	work done by compression space		
V_{se}	expansion swept volume		
V_{sc}	compression swept volume		
V_{de}	expansion dead volume		
V_{dc}	compression dead volume		
A	heat transfer effective surface		
A_{wg}	wetted area of the metal net of the regenerator		
l_h	heater length		
l_k	cooler length		
l_r	regenerator length		
V_r	heater volume		
V_h	cooler volume		
V_k	regenerator volume		
f	friction factor		
d_h	heater hydraulic diameter		
d_k	cooler hydraulic diameter		
d_r	regenerator hydraulic diameter		
C_V	specific heat at constant volume		
C_p	specific heat at constant pressure		
G	working fluid mass flow		
k	thermal conductivity		

Subscripts

rh	interference between regenerator and heater
kr	interference between cooler and regenerator
he	interference between heater and expansion chamber
ck	interference between compression chamber and cooler
h	heater
k	cooler
r	regenerator
c	compression chamber
e	expansion chamber
J	number of the inequality constraints
K	number of the equality constraints

Greek letter

ε	effectiveness of the regenerator
η	efficiency
ϕ	crank angle
γ	specific heat ratio
μ	dynamic viscosity

optimization [13–15]. The solution rout of the multi-objective optimization cases is a highly achieved target which requires the simultaneous satisfaction of a number of different and conflicting objectives. Evolutionary algorithms (EA) were applied in an attempt to stochastically solve problems of this generic class during the 18th century [16]. Inquiring a set of solutions of which each satisfies the objectives at an acceptable level without being overcome by any other solution, is a reasonable solving method to a multi-objective quandary [17]. A possibly uncountable collection of solutions so-called Pareto frontier is universally represented by the multi-objective optimization. The evaluated vectors of the Pareto frontier illustrate the best feasible trade-offs in the objective function space. In this case, multi-objective optimization of various thermodynamic and energy systems have been of high interest attention of researchers these days [18–25].

For considering all the above-mentioned issues, we have studied two objective functions including the power loss due to pressure drop into the heat exchangers and the efficiency. In addition, the multi-objective optimization is conducted with four decision variables including the temperature of working fluid at hot side, frequency, the stroke and the mean effective pressure.

Considering the parameters that have been neglected by most of the previous works on the performance optimization of Stirling engine such as the temperature of working fluid at hot side, frequency, the stroke and the mean effective pressure, we selected our approach for the current study. In the previous studies, the multi-optimization and adiabatic methods have not been linked together for Stirling engines and the adiabatic systems have only been utilized for modeling. In this work, by implementing the multi-objective optimization algorithms, the power loss due to pressure drop into the heat exchangers and the efficiency are minimized and maximized, respectively.

2. Theoretical model

In the constant temperature of the Schmidt analysis, it is assumed that the compression and the expansion spaces during the engine performance remain under constant temperature conditions [26]. This assumption provides a contradiction that the heater, the regenerator, and the cooler do not contribute in the any net heat transfer engine cycle. For this reason, they seem to be of none use. The real engine working spaces tend to the adiabatic mode during the processes. This assumption requires that the net heat transfer during a cycle is supplied by heat exchanger. An adiabatic model is shown in Fig. 1 for better understanding. This model is composed of five parts which are connected together sequentially. In this model, the working fluid in the cooler and the heater are at constant temperature conditions namely T_k and T_h , respectively. Also, the wire matrix regenerator and the embedded fluid are placed in a linear distribution. In this model, the assumption is that the working spaces are at adiabatic conditions. Also, it is assumed that there is no leakage of the working fluid and the pressure drop is not created in the fluid flow path. The performed work under these assumptions, is dependent on the changes in the volume of the compression and the expansion spaces and heat is transferred between the outside and the working fluid in the cooler and the heater which are Q_k and Q_h , respectively.

The most common method to obtain the required differential equations and the heat transfer in the heater and the cooler, is writing the energy equation and the ideal gas equation for each control volume. The assumed control volume is shown in Fig. 2

With getting differential of the mass balance for the assumed system [7], Eq. (1) is obtained.

$$Dm_c + Dm_k + Dm_r + Dm_h + Dm_e = 0 \quad (1)$$

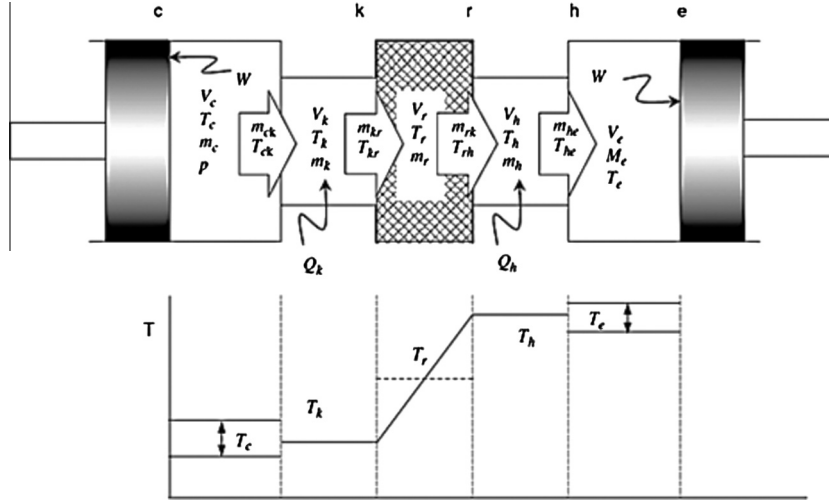


Fig. 1. The ideal adiabatic model [27].

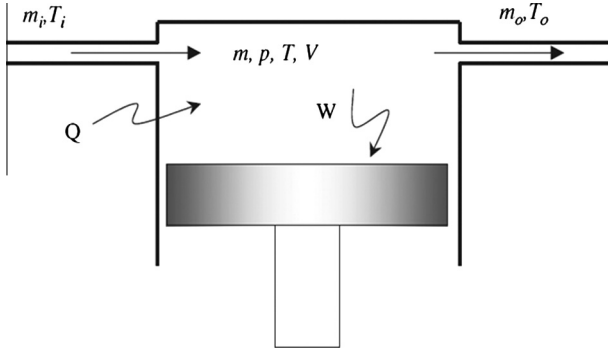


Fig. 2. Schematic of control volume [27].

By taking the logarithm of both sides of the equation of state, and taking differential from it and assuming the constant temperature and volume of the heat exchanger, the following equation can be written as Eq. (2).

$$\frac{Dp}{p} = \frac{Dm}{m} \quad (2)$$

By inserting Eq. (2) in Eq. (1), we have:

$$Dm_c + Dm_e + Dp \left(\frac{m_k}{p} + \frac{m_r}{p} + \frac{m_h}{p} \right) = 0 \quad (3)$$

However, the equation of energy can be written for space compression as followings:

$$DQ_c + c_p T_{ck} gA_{ck} = DW_c + c_v D(m_c T_c) \quad (4)$$

Since the compression space is assumed to behave adiabatically, hence $DQ_c = 0$ and the done work is $DW_c = pDV_c$. According to the law of mass conservation, the working fluid accumulation rate (Dm_c) is equal to the input mass to the control volume ($-gA_{ck}$).

$$c_p T_{ck} Dm_c = pDV_c + c_v D(m_c T_c) \quad (5)$$

Thus, the accumulation rates of the fluid in the expansion space and compression space are stated as:

$$Dm_c = \frac{pDV_c + \frac{c_v Dp}{\gamma}}{RT_{ck}} \quad (6)$$

$$Dm_e = \frac{pDV_e + \frac{c_v Dp}{\gamma}}{RT_{he}} \quad (7)$$

By inserting Eqs. (6) and (7) in Eq. (3), the desired equation based on the pressure differential is obtained.

$$Dp = \frac{-\gamma p \left(\frac{DV_c}{T_{ck}} + \frac{DV_e}{T_{he}} \right)}{\left[\frac{V_c}{T_{ck}} + \gamma \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} \right) + \frac{V_e}{T_{he}} \right]} \quad (8)$$

The changes in the volumes of the expansion and the compression spaces are defined as follows:

$$V_e = 0.5 \times V_{se} \times (1 + \cos(\theta)) + V_{de} \quad (9)$$

$$V_c = 0.5 \times V_{sc} \times (1 - \cos(\theta)) + 0.5 \times V_{sc} \times (1 + \cos(\theta - \theta)) + V_{dc} \quad (10)$$

Also, the temperatures of T_{ck} and T_{he} are determined by considering the fluid flow direction (gA). In fact, the direction of fluid flow can be assigned as followings:

$$Dm = gA_i - gA_o = m_i - m_o \quad (11)$$

Using Eq. (11) for each of the sectors of Fig. 1, the following equation is obtained:

$$m_{ck} = -Dm_c \quad (12)$$

$$m_{kr} = m_{ck} - Dm_k \quad (13)$$

$$m_{rh} = m_{kr} - Dm_r \quad (14)$$

$$m_{he} = m_{rh} - Dm_h \quad (15)$$

The total produced work by the engine can be writing as the sum of the work done in the compression and the expansion spaces:

$$DW = p(DV_c + DV_e) \quad (16)$$

By replacing energy equation, employing the aforementioned equations, and imposing some simplifications, we reach to more appropriate form of the energy equation.

$$DQ + (c_p T_i m_i - c_p T_o m_o) = \frac{c_p p DV + c_v V Dp}{R} \quad (17)$$

Work is not done inside the heat exchanger because the relative volumes remain constant. The following equations for the heater, the cooler and the regenerator can be written:

$$DQ_k = \frac{c_v V_k Dp}{R} - c_p (T_{ck} m_{ck} - T_{kr} m_{kr}) \quad (18)$$

$$DQ_r = \frac{c_v V_r Dp}{R} - c_p (T_{kr} m_{kr} - T_{rh} m_{rh}) \quad (19)$$

$$DQ_h = \frac{c_v V_h Dp}{R} - c_p (T_{rh} m_{rh} - T_{he} m_{he}) \quad (20)$$

2.1. The loss power due to pressure drops in heat exchangers

In Stirling engine, the friction of the fluid is related to the flow of gas through the heat exchanger and the pressure drop and this problem leads to reducing the output power. The Pressure drops can be calculated using the friction factor coefficient related to the Reynolds number. The pressure drop through the regenerator is calculated as followings [11,28]:

$$dP_R = \frac{2 \times f \times \mu \times V_r \times G \times l_r}{m_r \times d_r^2} \quad (21)$$

where f , V_r , G , l_r and d_r are the friction factor correlation, the regenerator volume, the working fluid mass flow, the regenerator length and the regenerator hydraulic diameter, respectively. The friction factor correlation has the following form for regenerator.

$$f = 54 + 1.43 \text{Re}^{0.78} \quad (22)$$

The pressure drop through the heater [28]:

$$dP_H = \frac{2 \times f \times \mu \times V_h \times G \times l_h}{m_h \times d_h^2} \quad (23)$$

$$f = 0.0791 \text{Re}^{0.75} \quad (24)$$

The pressure drop through the cooler [28]:

$$dP_K = \frac{2 \times f \times \mu \times V_k \times G \times l_k}{m_k \times d_k^2} \quad (25)$$

The friction factor correlation of cooler is the same as what was applied for heater.

The total pressure drop is calculated as:

$$dP = dP_R + dP_H + dP_K \quad (26)$$

By placing the total pressure drop in the following equation, the amount of lost power in the heat exchangers is obtained:

$$\text{power}_{\text{loss}} = \left(\int (dP \times dV) \right) \times f_r \quad (27)$$

2.2. Energy loss due to external conductivity

The energy loss due to the external conductivity is formed due to the non-ideal heat regenerating of the working fluid, within passing through the regenerator. In the non-ideal heat regenerator, the energy stored by the regenerator during the gas transmission from the expansion chamber to the compression chamber is not returned to the working fluid, completely [7]. In the case of ideal heat regenerator, the coefficient ε is equal to one. Energy loss due to external conductivity would be written as:

$$Q_{\text{loss}} = (1 - \varepsilon) \times (Q_{\text{max}} - Q_{\text{min}}) \quad (28)$$

The ideal regenerator coefficient is obtained from the following equation [29]:

$$\varepsilon = \frac{\text{NTU}}{\text{NTU} + 1} \quad (29)$$

The effect of the non-ideal regenerator is defined as followings by using the number of the transfer units (NTU):

$$\text{NTU} = \frac{\text{St} \times A_{\text{wg}}}{2A} \quad (30)$$

where A_{wg} is the wetted area of the metal net of the regenerator in the exposure to the working fluid. Also, St is the Staunton number that can be considered as [30]:

$$\text{St} = \frac{0.46 \times \text{Re}^{-0.4}}{\text{Pr}} \quad (31)$$

where Prandtl number (Pr) is considered as 0.7 for the utilized working fluid. The actual thermal energy in the cooler and the heater spaces is calculated by the following relation:

$$Q_H = Q_h + Q_{\text{rloss}} \quad (32)$$

$$Q_K = Q_k - Q_{\text{rloss}} \quad (33)$$

Finally, the actual work of the Stirling engine is calculated by obtaining of the loss work due to the pressure drop in the heat exchangers. The actual work and the efficiency of Stirling engine are defined as below:

$$\text{actual work} = (W - dW_{\text{loss}}) \quad (34)$$

$$\text{actual heat} = QH \quad (35)$$

$$\eta = \frac{\text{actual work}}{\text{actual heat}} \quad (36)$$

A method for simultaneous solving of these equations is using their initial values inside them. Thus, the solution process starts in a full cycle having the initial values of the all variables and integrating them. This solution is carried out using the fourth order Runge-Kutta. The implemented engine for this research was the GPU-3 whose characterizations are demonstrated in Table 1. The GPU 3 engine is of the Beta type Stirling engine which has been described in details in the literature [31,32]. For the purpose of optimization, the theory of multi-objective optimization is described in next chapter.

3. Multi-objective optimization

In the multi-objective problems, due to a conflict between the goals, there is no single optimal solution. Thus, the method should search several optimal solutions that are called the Pareto set [33,34]. In the Pareto optimal series, by moving from one point (solution) to another, an objective function gets better and another objective function gets worse; because both solutions of the series are non-dominated relative to each other.

The general form of multi-objective problems is as follows [34]:

$$\begin{cases} \text{Minimize/Maximize } f_m(x) & m = 1, 2, \dots, M \\ \text{Subject to } g_j(x) \geq 0 & j = 1, 2, \dots, J \\ h_k(x) = 0 & k = 1, 2, \dots, K \\ X_i^{(L)} \leq X_i \leq X_i^{(U)} & i = 1, 2, \dots, n \end{cases} \quad (37)$$

Table 1
The GPU3 engine parameters.

Parameter	Value
Heat source T_H (K)	1200
Heat sink T_C (K)	288
Engine's frequency (Hz)	41.7
Mean pressure P (MPa)	4.13
Regenerator length (mm)	22.6
Wire diameter d_w (μm)	40
Regenerator diameter (mm)	22.6
Porosity	0.697
Thermal conductivity ($\text{Wm}^{-1} \text{K}^{-1}$)	16.6
Total mass of gas (kg)	0.001135
Displacer diameter (mm)	69
Number of regenerators per cylinder	8
Number of gauzes of the matrix	308
Working fluid	Helium

An x solution is a vector of n decision variables:

$$X = [X_1 X_2 \dots X_n]^T \quad (38)$$

Each decision variable is limited between the amounts of the lower limit ($X_i^{(L)}$) and the upper limit ($X_i^{(U)}$). These limits make the space of the decision variable. Also, f is a vector of m objective functions.

$$f(x) = [f_1(x) f_2(x) \dots f_n(x)]^T \quad (39)$$

Of which $g_j(x)$ and $h_k(x)$ are constraint function that J and K are number of the inequality constraints and number of the equality constraints, respectively. Although the single-objective optimization problems may have a unique optimal solution, the multi-objective optimization problems usually have a possibly uncountable set of solutions on a Pareto front. Each solution associated with a point on the Pareto front is a vector whose components represent trade-offs in the decision space or Pareto solution space. The multi-objective optimization problems evaluation function, $f: \Omega \rightarrow \Lambda$ maps decision variables ($\mathbf{x} = x_1, \dots, x_n$) to vectors ($\mathbf{y} = a_1, \dots, a_k$). This situation is represented in Fig. 3 for the case $n = 2$, $m = 0$, and $k = 3$. This mapping may or may not be onto some region of objective function space, dependent upon the functions and constraints composing the particular multi-objective optimization. [35]

3.1. NSGA-II

NSGA-II is the second version of the “Non-dominated Sorting Genetic Algorithm” for solving non-convex and non-smooth single and multi-objective optimization problems. The concepts of

dominance and crowding distance are important subjects in the NSGA-II which were widely described by Deb et al. [37,38]

Fig. 4 shows the performance of NSGA-II multi-objective optimization model. In this method, the children population (Q_t) is made up by the parent population (P_t). Then two populations combine with each other and (R_t) population of size $2N$ is composed. Then, non-dominating sorting is used to classify the entire of the (R_t) population. Following the creating of different non-dominance ranks ($F_1, F_2, \dots, F_l$), the next population is filled in order of the priority of these ranks. The next population (P_{t+1}) starts to fill the best non-dominance ranks, then the second non-dominance ranks and so on, until the P_{t+1} is filled. About the last rank, the solutions, which have more crowding distance, are the priority to fill P_{t+1} . Finally, the population of children (Q_{t+1}) from the parent population (P_{t+1}) is created using a select tournament function and the operators of crossover and mutation.

3.2. Objective functions, decision variables and constraints

A mathematical model of the optimization process has three general parts (the objective functions, decision variables and constraints problem) that are described here in. Two objective functions for this study are the power loss due to pressure drop inside the heat exchangers and the efficiency of the Stirling engine. In this paper, four decision variables have been considered as followings:

- f_r : engine's frequency,
- p : mean effective pressure,
- S : stroke,
- T_h : the temperature of working fluid at hot side.

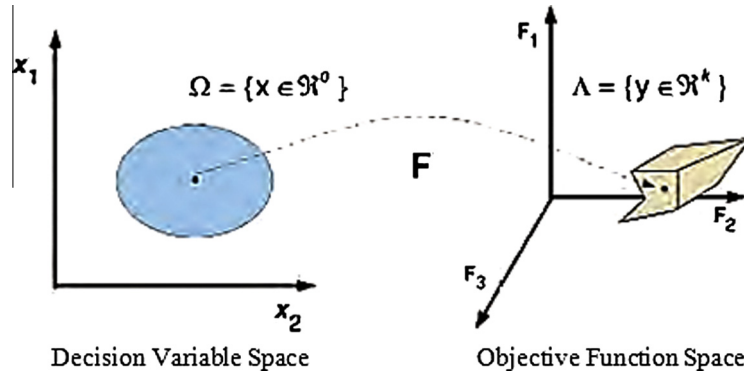


Fig. 3. A schematic of the decision variable space and objective function space [36].

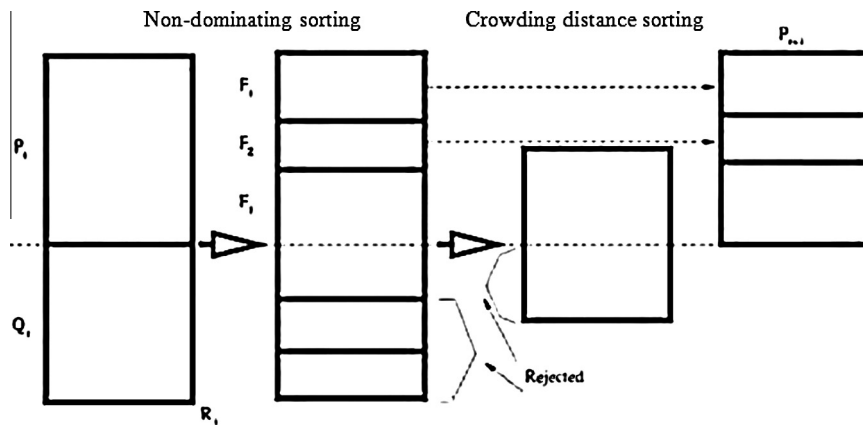


Fig. 4. The structure of NSGA-II multi-objective optimization model [36].

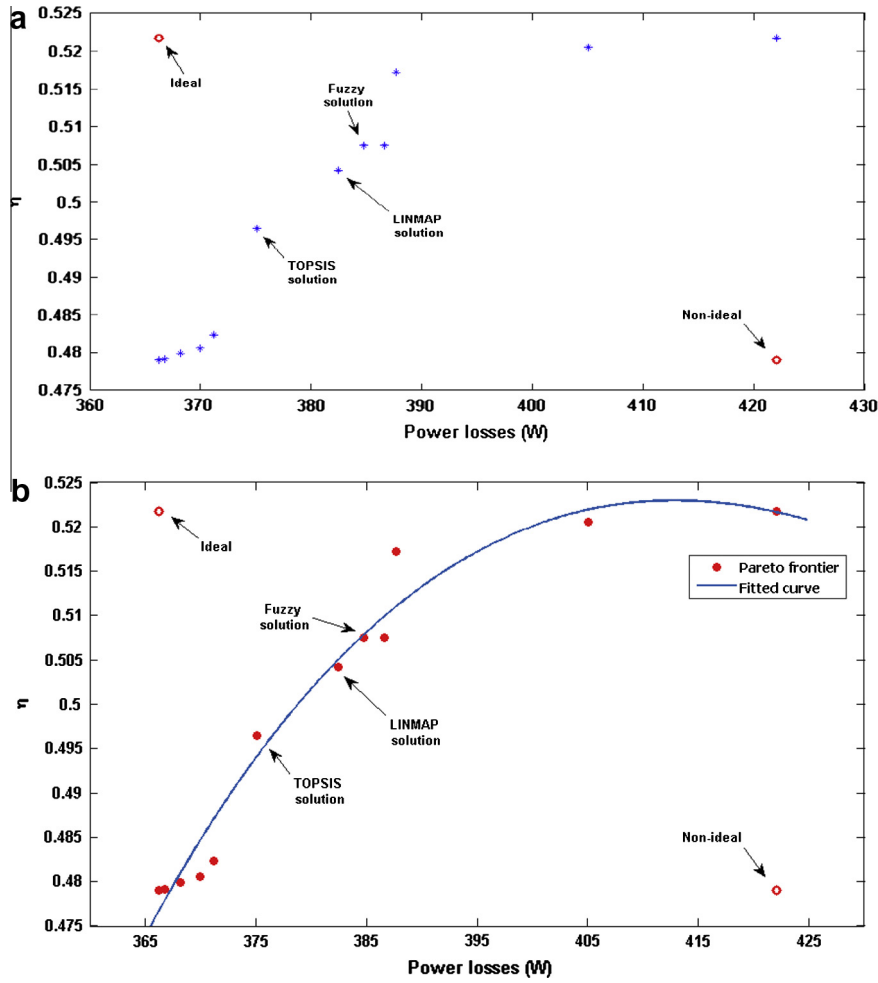


Fig. 5. (a and b) Pareto optimal frontier in the objectives' space.

The objective functions with respect to the following constraints have been solved:

$$\begin{cases} 40 \leq f_r \leq 50 \text{ Hz} \\ 4.1 \leq p \leq 6.89 \text{ MPa} \\ 0.03 \leq S \leq 0.07 \text{ m} \\ 900 \leq T_h \leq 1100 \text{ K} \end{cases} \quad (40)$$

For choosing the optimum point of the engine, a decision should be taken and there are different ways to make decisions. In this work, three methods including TOPSIS, LINMAP, and Fuzzy have been used which are explained briefly as followings. In the LINMAP method, the optimal point is a point on the Pareto Front that has the shortest distance to the ideal point. The ideal point is a point that has best situation. In the TOPSIS method, the optimal point is a point which has the lowest value of C_L .

$$C_L = \frac{d_{i+}}{d_{i+} + d_{i-}} \quad (41)$$

where d_{i+} and d_{i-} are the distance of each point from the positive ideal point and negative ideal point, respectively. In the Fuzzy method, the Fuzzy Bellman-Zadeh is used which is explained in further details in the literature [39–42].

4. Results and discussion

The power loss due to pressure drop into the heat exchangers and the efficiency of Stirling engine are minimized and maximized,

respectively. These objectives were maximized simultaneously using a class of multi-objective evolutionary algorithms called NSGA-II. The optimization process was performed using objective functions expressed by Eqs. (27) and (36) along with the constraints which are expressed in Eq. (40). Fig. 5a illustrates the Pareto frontier in the proposed objectives' space obtained in the optimization scenario. Three final solutions were selected by the Fuzzy Bellman-Zadeh, LINMAP and TOPSIS decision-making methods which are indicated in this figure. According to Fig. 5a, the obtained points by LINMAP and TOPSIS are approached towards each other. Also, it was shown that the optimal value of the power loss varied from 365 W to 425 W and the optimal value of η was between 0.475 and 0.52. Based on the obtained curve fit depicted in Fig. 6b, Eq. (42) was developed.

$$\eta = a \times \exp(b \times \text{powerloss}) + c \times \exp(d \times \text{powerloss}) \quad (42)$$

The coefficients (with 95% confidence bounds) and goodness of fit are summarized in Table 2.

Table 2
The coefficients and goodness of fit.

Coefficient			
$a = 19.33$	$b = -0.00631$	$c = -55.22$	$d = -0.009958$
Goodness of fit			
SSE: 0.0001045	R-square: 0.9709	Adjusted R-square: 0.9611	RMSE: 0.003407

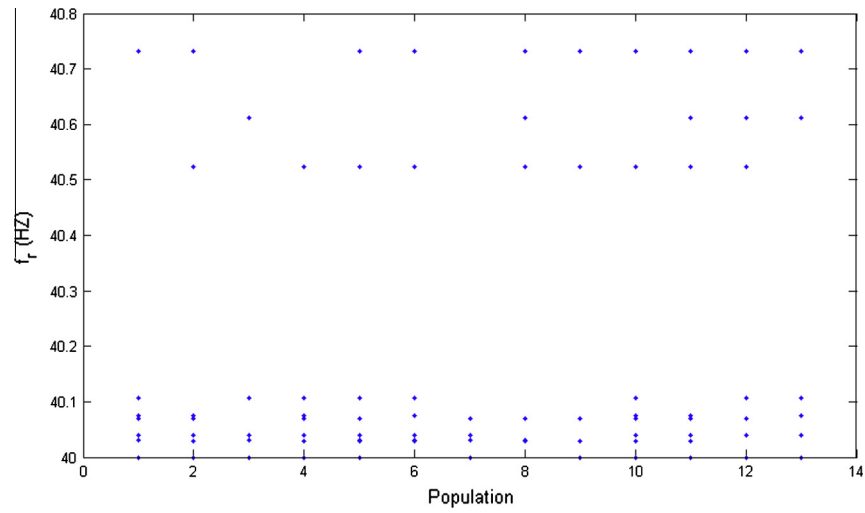


Fig. 6. The distribution of f_t for the optimal points on Pareto front.

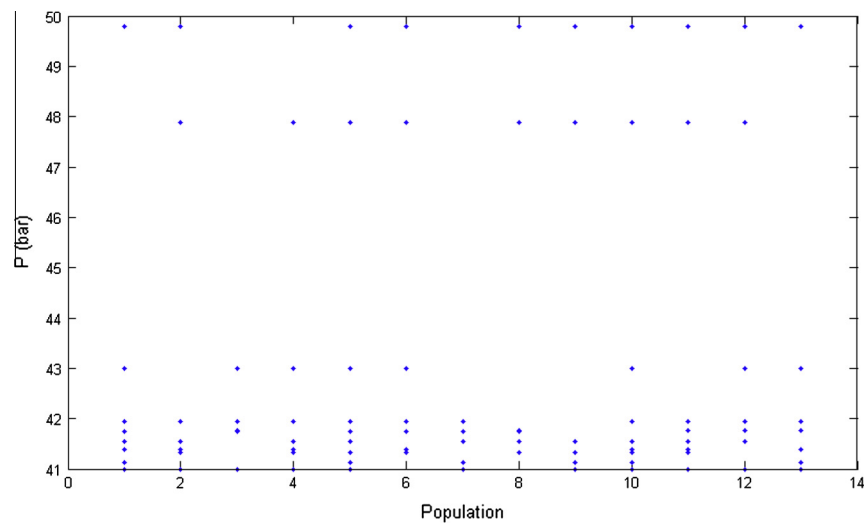


Fig. 7. The distribution of P for the optimal points on Pareto front.

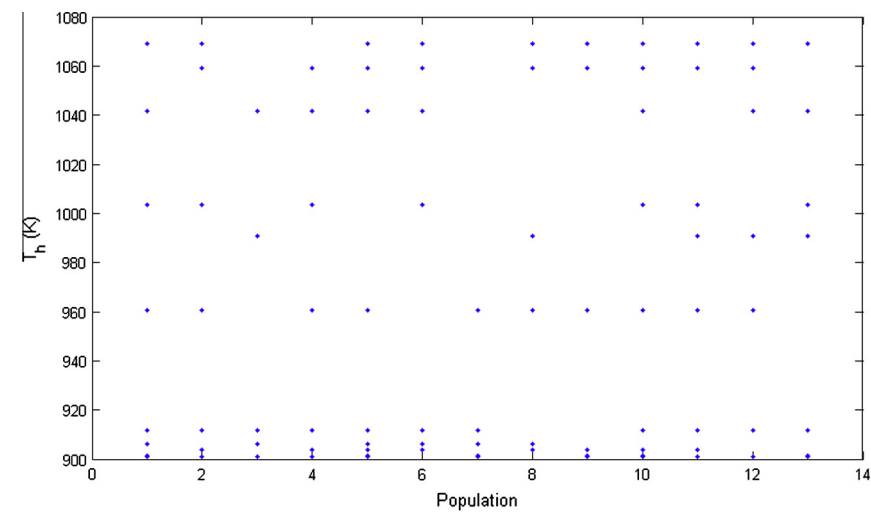


Fig. 8. The distribution of T_h for the optimal points on Pareto front.

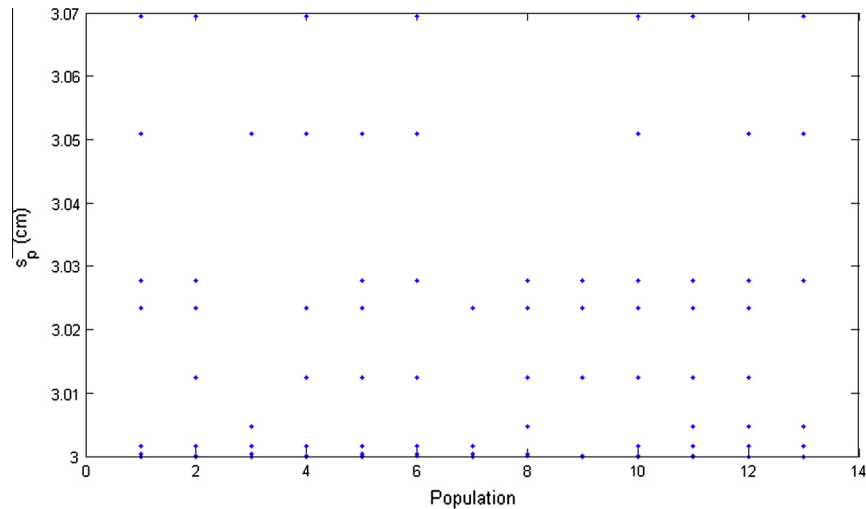


Fig. 9. The distribution of S_p for the optimal points on Pareto front.

Table 3

Comparison of the final optimal solutions of this paper (multi objective-multi variable) with corresponding results.

Method	T_h (K)	S (m)	P (bar)	f_r (Hz)	Power loss (W)	η (%)
Initial theoretical status	939.30	4.6	41.3	41.7	441.759	48.73
TOPSIS	1069.1	3.03	49.78	40.73	375.1	49.65
LINMAP	903.9	3.00	41.32	40.03	382.4	50.42
Fuzzy	1059.2	3.01	47.88	40.53	384.8	50.75
Urieli and Berchowitz [7]	–	4.6	41.3	41.7	–	62.06
Timoumi dynamic model without losses [43]	–	4.6	41.3	41.7	–	54.96
Timoumi dynamic model with loss dissipation (M1) [44]	–	4.6	41.3	41.7	–	53.10
Timoumi dynamic best model [44]	–	4.6	41.3	41.7	–	52.50
Experiment [44]	–	4.6	41.3	41.7	–	35.00

Table 4

Error analysis based on the mean absolute percent error (MAPE) method.

Decision-making method	Fuzzy		TOPSIS		LINMAP	
Objectives	Power loss	η	Power loss	η	Power loss	η
Average error (%)	17.16	1.92	2.44	0.06	4.90	2.10

Figs. 6–9 demonstrate the scattered distribution of the design variables versus population. These figures can provide a good vision on the variation of the decision variables on the Pareto frontier. Distribution of variables for the optimum points on the Pareto front for f_r over population are exhibited in Fig. 6 and the range of f_r varied from 40 to 40.8. It can be seen from Fig. 7 that the magnitude of P for optimal Pareto front varied from 41 to 50 bar. As shown in Fig. 8, the variation of T_h for the optimal Pareto front varied from 900 K to 1080 K. Finally, the variables distributions for the optimum points on Pareto front for S_p over the population are exhibited in Fig. 9. The upper and lower ranges of S_p are positioned from 3.05 to 3.00.

The optimal results for the decision parameters and objective functions using LINMAP, TOPSIS and Bellman-Zadeh decision-making methods are summarized in Table 2. The comparison of the optimal results obtained in this paper using three decision making methods with the corresponding results obtained in Refs. [7,43,44] is demonstrated in Table 3. Also, the initial status in where the parameters had not been optimized yet, are reported in the table. In this state, the code was written and the model was obtained, without considering optimization. With multi-objective optimization, we could decrease the power loss caused by the pressure drop in the system.

According to Table 3, the characteristics of this work are compared to the other works which have not considered the important losses. This reason caused that they reported higher efficiency. Also, if the efficiency is optimized as single-objective, the efficiency will be higher than the reported efficiency while this work has been carried out as two-objective optimization.

The mean error analyses for the objective functions are categorized in Table 4. The analysis is done for all mentioned decision-making methods.

5. Conclusions

In this study, adiabatic analysis has been applied to determine the power loss and the efficiency of a Stirling engine. The power loss and the efficiency of the Stirling engine were considered simultaneously for multi-objective optimization and the engine's frequency (f_r), mean effective pressure (p), stroke (S_p) and the temperature of the working fluid of hot side (T_h) were considered as the design parameters. Multi-objective evolutionary algorithm is considered based on NSGA-II algorithm and the Pareto optimal frontier in the objectives space was obtained. A final optimal solution was selected from the solutions of the Pareto frontier using three decision making methods including LINMAP, TOPSIS and

Fuzzy methods. It was shown that the multi-objective multi-variable approach leads to more desirable design of the Stirling engine if it is compared with the single-objective single-variable approaches which have been performed in the previous researches. Also, with comparing the results, the TOPSIS decision making method leaded to a better conclusion in aspect of power loss.

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